PERFORMANCE OF UNITARY SPACE-TIME MODULATION IN RAYLEIGH FADING

Christian B. Peel and A. Lee Swindlehurst

Brigham Young University
Electrical and Computer Engineering Dept.
459 CB, Provo, UT 84602
chris.peel@ieee.org, swindle@ee.byu.edu

ABSTRACT

Rapid temporal variations in fading wireless channels pose a significant challenge for space-time modulation and coding algorithms. In this paper, we examine the performance degradation that results when fast fading is encountered. We consider two models. First, we examine a constant channel of arbitrary length that experiences random variations between frames (as might occur in a time-division multiple access system). Second, we adopt a model in which the channel is constant over each space-time symbol period, but varies between symbols according to a first-order AR model. For both cases, we find a performance floor at high SNR for both trained and differential unitary space-time modulation. Though trained modulation provides an advantage at low SNR, differential coding can provide a significant advantage at high SNR where the effects of the changing channel dominate. Simulation results are provided to support our analysis.

1. INTRODUCTION

Communication systems with multiple transmit and receive antennas promise high data rates with low probability of error [1, 2]. For the single antenna case at high signal-to-noise ratio (SNR), capacity increases one bit/s/Hz per three db increase in SNR. In contrast, for M transmit and N receive antennas, the capacity increases $\min(M,N)$ bits/s/Hz per three db increase in SNR. This exciting result is derived under the assumption that the receiver knows the Rayleigh fading coefficient between each transmit and receive antenna, and that they are independently distributed.

Knowledge of the channel coefficients at the receiver is a non-trivial assumption; typically a training signal is sent from which the channel is estimated and used for decoding subsequent symbols, until the channel has changed enough to require training again. The number of channel uses T_C over which the channel is approximately constant is known as the *coherence interval.* As the number of antennas used and the speed of fading increase, the fraction of the coherence interval that must be used for training increases. This obviously decreases the available data rate, and motivates interest in schemes that do not require explicit knowledge of the channel coefficients at the receiver.

Marzetta and Hochwald have studied situations where neither the transmitter nor receiver know the channel [3]. Assuming piecewise-constant channel coefficients, they proposed signal constellations composed of unitary matrices as a means to achieve capacity at high SNR. These can be seen as multiple-antenna generalizations of phase-shift keying (PSK) for scalar channels. Hughes [4] and Hochwald et al. [5] apply these signals to the unknown channel by extending differential phase-shift keying (DPSK) ideas to the multiple antenna case. Tarokh also discusses differential modulation with orthogonal signals in [6].

The piecewise-constant model for the time-varying channel coefficients assumed in these papers is useful for several reasons. It accurately describes the way a channel might appear in a time-division multiple access or frequency-hopping system, and its effects are simple to analyze. In other applications, however, its inability to account for the memory of the channel make it less attractive. To incorporate channel memory into our analysis, we adopt a first-order auto-regressive (AR) model for the time-variations of the channel coefficients.

We first analyze the high-SNR performance of unitary space-time coding [3, 5] with the piecewise-constant assumption. Then the AR model of the time-varying coefficients is used to accurately describe continuous fading, and provide an explicit SNR ceiling beyond which increasing power provides no benefit. For $M = N = 1$, our high-SNR error floor reduces to that of Korn [7]. Our analysis is based on the use of unitary signal matrices [3], which allows us to combine the AR input with the additive noise of the channel equation to effectively create a noise term with higher power.

2. BACKGROUND

In this section we describe block unitary space time coding over the standard Rayleigh fading channel. We then discuss application of a first-order AR process to models of the mobile fading channel. In what follows, we let $\mathcal{CN}(0,1)$ denote a zero-mean, unit-variance, circularly symmetric complex Gaussian distribution. We will call a matrix *isotropically distributed* (i.d.) if its elements are $\mathcal{CN}(0,1)$ random variables.

2.1. Unitary Space-Time Coding for Rayleigh Fading

This work applies to a Rayleigh flat-fading communications environment with multiple transmit and receive antennas. Each transmit antenna is connected with each receive antenna through an independent complex channel coefficient. These channel coefficients are assumed to be independent from element to element across the antenna array, but dependent with time. At each receive antenna, interference and other disturbances add temporally and spatially independent noise to the signal.

This work was supported by the National Science Foundation under Wireless Initiative Grant CCR 99-79452.

We formalize these statements using notation similar to that in [5]: for $m = 1, \ldots, M$ transmit, and $n = 1, \ldots, N$ receive antennas, at time instants $t = 1, 2, \ldots, T$, the channel coefficient is $h_{m,n,t} \sim \mathcal{CN}(0,1)$. The noise $w_{t,n}$ is also $\mathcal{CN}(0,1)$. For a transmitted signal $s_{t,m}$, the signal at each receive antenna is written

$$x_{t,n} = \sqrt{\frac{\rho}{M}} \sum_{m=1}^{M} h_{m,n,t} s_{t,m} + w_{t,n}. \quad (1)$$

The values in this expression are normalized so that ρ represents the SNR expected at each receive antenna, and does not depend on the number of transmit antennas.

The channel equation in (1) allows for different channel coefficients at every time instant. One common simplification is to assume that the channel is constant for T consecutive samples, and express the operation of the channel in matrix form:

$$X_\tau = \sqrt{\frac{\rho}{M}} S_\tau H_\tau + W_\tau, \quad (2)$$

where X_τ and W_τ are $T \times N$ matrices constructed from $x_{t,n}$ and $w_{t,n}$, S_τ is a $T \times M$ matrix constructed from $s_{t,m}$, and H_τ is a $M \times N$ matrix formed from $h_{m,n}$. We refer to S_τ as the space-time symbol transmitted at time τ, and the subscript on H_τ indicates that the channel will, in general, be different from symbol to symbol. Modulation and performance using this model are discussed in [3] and [8] for both known and unknown channels.

A common assumption is that the receiver knows the channel. This knowledge is usually obtained by transmitting known symbols, which the receiver uses to estimate the channel. This estimate is then used for decoding $K-1$ subsequent symbols over which the channel is assumed to be constant. We will refer to this as trained, or known channel modulation.

One method of modulation with an unknown channel is differential unitary space-time modulation [4, 5]. This scheme uses data at the current and previous time instants for encoding and decoding. The channel matrices are assumed equal at times τ and $\tau - 1$ and are denoted without subscript by H. The current signal matrix is a unitary rotation of the previous signal; $S_\tau = V_{z_\tau} S_{\tau-1}$, where $z_\tau \in 0, \ldots, L-1$ indexes the unitary constellation matrix to be transmitted. Using these definitions, and working with the current received data $\mathcal{X}_\tau$, the following expressions are obtained in [5]:

$$\begin{aligned} \mathcal{X}_\tau &= \sqrt{\frac{\rho}{M}} V_{z_\tau} S_{\tau-1} H + W_\tau + (1-1) V_{z_\tau} W_{\tau-1} && (3) \\ &= V_{z_\tau} \mathcal{X}_{\tau-1} + W_\tau - V_{z_\tau} W_{\tau-1} && (4) \\ &= V_{z_\tau} \mathcal{X}_{\tau-1} + \sqrt{2} \hat{W}_\tau \ . && (5) \end{aligned}$$

In (3), $V_{z_\tau} W_{\tau-1}$ is added and subtracted from (2), resulting in (4) which does not explicitly depend on H. Finally, because the noise matrices are statistically invariant to multiplication by unitary matrices, (5) is obtained, which is called the "fundamental differential receiver equation" in [5].

Because the effective channel ($\mathcal{X}_{\tau-1}$) has signal strength ρ, the system has an effective signal to noise ratio of $r = \rho/2$. This factor of 2 is the multiple-antenna analog of the well-known three db loss in performance when using DPSK versus coherent PSK.

2.2. Auto-regressive Fading Channel Model

In Section 4, we will analyze the performance of differential unitary space-time modulation under the assumption that the channel matrix varies according to the following first-order AR model:

$$H_\tau = \sqrt{\alpha} H_{\tau-1} + \sqrt{1-\alpha} E_\tau \ , \quad (6)$$

where H_0 and E_τ are i.d., E_τ is independent from symbol to symbol, $\alpha \in \mathbb{R}$ and $0 \le \alpha \le 1$. Under this model, each element of H_τ has zero mean and unit variance.

The one-step AR model in (6) can be extended to predict K steps in the future as follows:

$$H_\tau = \sqrt{\alpha^K} H_{\tau-K} + \sqrt{1-\alpha^K} \tilde{E}_\tau \ .$$

However, in this model, $\tilde{E}_\tau$ is temporally correlated, a fact that would considerably complicate our analysis. Instead, we choose a simpler approach in which the channel is modeled using an explicit K-step predictor for each K:

$$H_\tau = \sqrt{\alpha_K} H_{\tau-K} + \sqrt{1-\alpha_K} \hat{E}_\tau \ , \quad (7)$$

where the elements of $\hat{E}_\tau$ are $\mathcal{CN}(0,1)$ and temporally white. The only difficulty with this approach is that some method must be used to choose α_K in a reasonable way. We solve this problem by matching the autocorrelation function of the AR process to that predicted using Jakes' well-known model [9].

Under Jakes' model, the autocorrelation function of a fading process $h_{m,n,\tau}$ is $r_{hh}(t) = J_0(2\pi f t)$, where $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind, $f = f_d T_s$, f_s is the maximum Doppler frequency in the fading environment, and T_s is the sampling period. Solving the Yule-Walker equations for α_K in the first-order AR process (7) gives

$$\alpha_K = \left[\frac{r_{hh}(TK)}{r_{hh}(0)} \right]^2 = J_0(2TK\pi f)^2 \ . \quad (8)$$

The AR model is a very good approximation to Jakes' model for realistic (small) values of f. This fact is borne out by the simulation results of Section 5, where excellent agreement was obtained with data generated according to Jakes' model, but analyzed with the AR model using (8).

3. PERFORMANCE FOR THE PIECEWISE-CONSTANT CHANNEL

The piecewise-constant channel model provides a good description of the time-division multiple access and frequency hopping channels, and is useful even for slowly fading processes [8]. Note that in [5] and [8], T represents the length of the coherence interval, as well as the length of the space-time codes used. In our development below, T will be used to indicate the length of the codes used, while T_C will indicate the coherence interval of the channel. In general, we will allow $T \neq T_C$.

There are two sources of error for space-time modulation in the Rayleigh flat-fading channel: channel time variations and additive noise. At high SNR, the probability of error will be dominated by the variation of the channel. If the piecewise-constant model holds exactly and the SNR is high enough, the only time an error will occur is at the transition between one coherence interval and another.

Figure 1 illustrates how training and errors occur with the piecewise-constant channel when the SNR is high. We assume that the channel is estimated during the first symbol of a K-symbol frame (recall that each "symbol" is actually a $T \times M$ matrix of

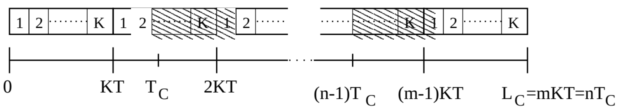

Fig. 1. Errors on the high-SNR piecewise-constant channel.

transmitted signal data). We are interested in the probability that the Kth symbol after the start of training is in error. If there are L symbols in our signal constellation, then the symbols from time T_C (when the channel changes) until the next training interval are assumed to be in error with probability $(L-1)/L$ (symbols in error are shown as shaded in the figure). These errors occur periodically with period L_C, the least common multiple of T_C and KT. The Kth symbol is possibly in error $n-1$ times, where $n = L_C/T_C$, over a total of $m = L_C/(KT)$ symbols, for a probability of error of

$$P_e = \frac{n-1}{m} = \frac{L-1}{L} KT \left(\frac{1}{T_C} - \frac{1}{L_C} \right) .$$

If KT divides T_C then the training exactly matches the coherence interval, and there will be no errors due to the channel. If $K = 2$, then when T_C is odd, $P_e = \frac{(L-1)T}{LT_C}$, which (as we will see in the next section) is the probability of error for differential coding. One interesting fact is that while the probability of error at low SNR depends on on the number of receive antennas, it does not at high SNR.

For differential unitary space-time modulation at high SNR, an error occurs once a coherence interval, when the previous channel is not the same as the current channel. The number of symbols per coherence interval is T_C/T, thus the probability of error is $P_e = \frac{(L-1)T}{LT_C}$. Note that because the known channel must use training to obtain a channel estimate which it will use for K symbols, the high-SNR probability of error for the trained channel will be about K times larger than that for the unknown channel at the Kth symbol after training. This is in contrast to the three db advantage of trained modulation at low SNR.

4. PERFORMANCE FOR THE AUTO-REGRESSIVE TIME-VARYING CHANNEL

Methods that estimate the channel using training data implicitly assume that the channel is piecewise-constant; *i.e.,* it is assumed that the training-based channel estimate is "good" until the next training interval. Of course, no wireless channel is truly constant over any time period, and the longer the interval since the last training data was sent, the more the channel estimate will differ from the truth. To more accurately model the effects of a time-varying channel, we assume in this section that the channel is constant over each symbol, but varies between symbols according to the first-order AR model introduced in Section 2. The following theorem quantifies the reduction in *effective SNR* (ESNR) that results under this time-varying channel model.

Theorem 1 (ESNR for trained modulation). *Given the data model of* (2)*, assume that the channel varies according to* (7) *with AR parameter* α_K*. If unitary space-time modulation is implemented with a training-based channel estimate, the effective SNR for the* K*th symbol after training is*

$$\rho_T(\rho, \alpha_K) = \frac{\rho \alpha_K}{1 + (1-\alpha_K)\rho\, T/M} \leq \rho \, , \tag{9}$$

where the subscript T *on* ρ_T *indicates* ***trained*** *modulation.*

Proof. The AR model of (7) is used to describe how the channel has changed since the training data was transmitted. Substituting this into the channel model (2) yields

$$X_t = \sqrt{\frac{\rho}{M}} S_t \left(\sqrt{\alpha_K} H_{t-K} + \sqrt{1-\alpha_K} \hat{E}_t \right) + W_t. \tag{10}$$

Because the statistics of an i. d. matrix are invariant to multiplication by a unitary matrix, we combine E_t and W_t by adding variances to obtain

$$X_t = \sqrt{\alpha_K \frac{\rho}{M}} S_t H_{t-K} + \sqrt{1 + \rho T/M(1-\alpha_K)} \hat{W}_t, \tag{11}$$

where $\hat{W}_t$ has the same statistical properties as W_t. By normalizing the equation so that the noise has unit variance, we obtain an expression from which the ESNR in equation (9) is easily identified:

$$\mathcal{X}_t = \sqrt{\frac{\rho/M \alpha_K}{1 + \rho T/M(1-\alpha_K)}} S_t H_{t-K} + \hat{W}_t. \tag{12}$$

□

Combining the effects of noise and channel time-variation into a single parameter provides a straightforward link with previous work that assumes a piecewise-constant channel. In essence, we can treat the time-varying case using a time-invariant channel model with a lower effective SNR. The probability of error expressions and maximum-likelihood decoders presented in [3, 5] can then be directly applied by simply replacing the true SNR ρ by the ESNR ρ_T.

Note that, in the limit as the channel becomes constant ($\alpha_K \to 1$), the ESNR converges from below to the original SNR due only to the additive noise, as desired:

$$\lim_{\alpha_K \to 1} \rho_T = \rho \, .$$

For a fast fading channel that varies randomly from symbol to symbol ($\alpha_K \to 0$), then $\rho_T \to 0$. As the SNR increases, the ESNR becomes a function only of the fading parameters, and is independent of ρ:

$$\lim_{\rho \to \infty} \rho_T = \frac{M}{T} \frac{\alpha_K}{1-\alpha_K} \, . \tag{13}$$

This confirms the intuition that as we increase the power to the system, errors due to thermal and other noise will become less significant, and performance will be dominated by errors induced by the changing channel.

The derivation of the differential receiver equation (5) in Section 2.1 assumes that the channel is constant for overlapping periods of $T = 2M$ time instants. A simple inductive argument can be used to show that if the channel is constant for two symbols at symbol τ, then at symbol $\tau+1$ it must have been constant for three symbols, and so on. Clearly there are limitations to this assumption: the only way that it can hold in practice is if the channel is in fact constant. In this section, we use the AR channel model to obtain a more realistic result, as outlined in the following theorem.

Theorem 2 (ESNR for differential modulation). *Given the data model of* (2), *assume that the channel varies according to* (7) *with AR parameter* α. *If differential unitary space-time modulation is implemented, the effective SNR is*

$$\rho_D(\rho,\alpha) = \frac{\alpha\rho}{1+\alpha+(1-\alpha)\rho T/M} \leq \frac{\rho}{2}\,, \tag{14}$$

*where the subscript D on ρ_D indicates **differential** modulation.*

Proof. Using (2) and (7) with AR parameter α, we can write

$$\mathcal{X}_{\tau-1} = \sqrt{\frac{\rho}{M}} S_{\tau-1} H_{\tau-1} + W_{\tau-1}$$

$$\mathcal{X}_{\tau} = \sqrt{\frac{\rho}{M}} V_{z_\tau} S_{\tau-1}\left(\sqrt{\alpha} H_{\tau-1} + \sqrt{1-\alpha} E_\tau\right) + W_\tau\ ,$$

where we have used $S_\tau = V_{z_\tau} S_{\tau-1}$ for differential modulation. After adding and subtracting $\sqrt{\alpha} V_{z_\tau} W_{\tau-1}$ we obtain

$$\begin{aligned}\mathcal{X}_\tau \;&=\; \sqrt{\alpha} V_{z_\tau}\left(\sqrt{\frac{\rho}{M}} S_{\tau-1} H_{\tau-1} + W_{\tau-1}\right) + W_\tau + \\ &\quad \sqrt{1-\alpha} V_{z_\tau}\sqrt{\frac{\rho}{M}} S_{\tau-1} E_\tau - \sqrt{\alpha} V_{z_\tau} W_{\tau-1}\ .\end{aligned}$$

Noting that the term in parentheses is $\mathcal{X}_{\tau-1}$ and that multiplication by unitary matrices does not change the statistics of the noise matrices, we obtain

$$\mathcal{X}_\tau = \sqrt{\alpha} V_{z_\tau} \mathcal{X}_{\tau-1} + \left(\sqrt{1+\alpha+(1-\alpha)\rho T/M}\right)\hat{W}_\tau. \tag{15}$$

Normalizing to obtain unit variance noise, and noting that the effective channel ($\mathcal{X}_{\tau-1}$) has signal power ρ, the result in (14) is established. □

Note that, as expected, $\lim_{\alpha\to 1}\rho_D = \rho/2$, and

$$\lim_{\rho\to\infty}\rho_D = \frac{M}{T}\frac{\alpha}{1-\alpha}. \tag{16}$$

Equation (15) is the analog of (5) in Section 2.1. As in the known channel case, the effective SNR ρ_D can be used in place of the true SNR in the probability of error expressions in [3] and in the capacity expressions derived in [8] for differential unitary space-time modulation.

5. SIMULATION RESULTS

We have presented analytic results quantifying performance for a continuously varying fading channel with memory, as well as performance in the high SNR region for a piecewise-constant channel. We now present simulation results that support our analysis. We focus on the probability of symbol error when using the diagonal codes presented in [5]. In particular, we will use $L = 2$ signals in our constellations which consist simply of the identity and negative identity matrices. Though our results hold true for all unitary signal matrices, we use these constellations because of the resulting simple analytic expressions for probability of error. In the figures that follow, a square or circle indicates a simulation result for that SNR value. We generated channel coefficients that obey the two channel models, and simulated them with six million symbols at each SNR value shown to calculate the probability of error results.

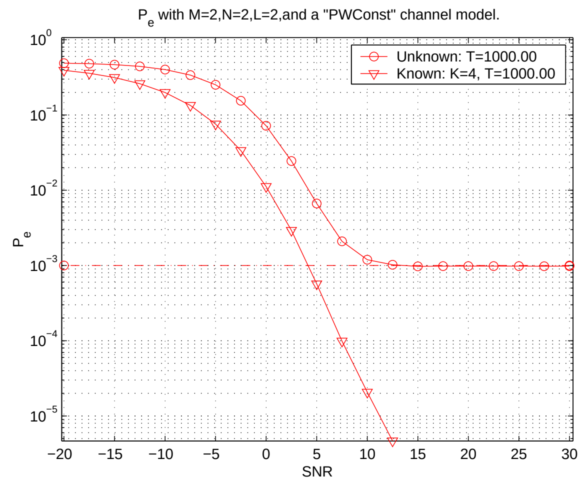

Fig. 2. Results for $M = 2, N = 2$ with piecewise-constant channel coefficients.

In Section 3, we noted that for the piecewise-constant model at high SNR, the performance of both trained and differential modulation depends primarily on the coherence period T_C, rather than on SNR. Specifically the probability of symbol error with an L symbol constellation as $\rho \to 0$ is $(L-1)/L$. For $L = 2$, $M = 2$, and $T = M$, the high SNR probability of error for trained and differential modulation is $K(1/T_C - 1/L_C)$ and $1/T_C$ respectively. In Figure 2 we see that these results hold true in simulation. In this figure, the solid lines interpolate the results of simulations, while the dashed line shows the high-SNR error floor for differential modulation. Note that for $K = 4$ the coherence interval of $T_C = 1000$ is divisible by $TK = 8$ so there is no error floor for trained modulation. In this case there is no point at which differential modulation gives better error performance than trained modulation.

In Section 4 we condensed the pertinent information from the continuously time-varying channel into an ESNR parameter which is a function of true SNR and the first-order AR coefficient of the time-varying channel model. Figure 3 illustrates our results for $M = 2$ transmit antennas, $N = 2$ receive antennas, training interval $K = 4$, and a fading parameter of $f = 0.03$. Use of the ESNR parameter with α_K from (8) in place of the true SNR in the probability of error expressions in [3] gives the analytic results for trained and differential modulation shown with the dashed lines. The solid lines give the results of simulations with channel coefficients which obey Jakes' model. Our analytic and simulation results match well for both differential and trained modulation.

Figure 4 shows performance of differential modulation with $M = 2$, $N = 1$, $K = 3$, and using Jakes' model with fading parameters $f = [0.05\ 0.02\ 0.01]$. We see that as the temporal variability of the channel increases, the high-SNR error floor also increases. In addition, the SNR above which performance is dominated by the channel decreases as the fading parameter increases, as suggested by (16). Our simulation and analysis match very well, with the largest difference at $f = 0.05$, where the assumption that the channel is constant over an entire symbol is less correct than at the slower fading parameters $f = [0.02\ 0.01]$.

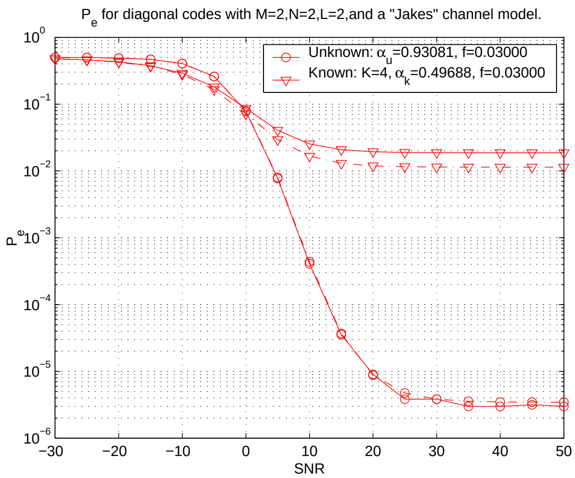

Fig. 3. Results for $M = 2, N = 2$ with Jakes channel coefficients.

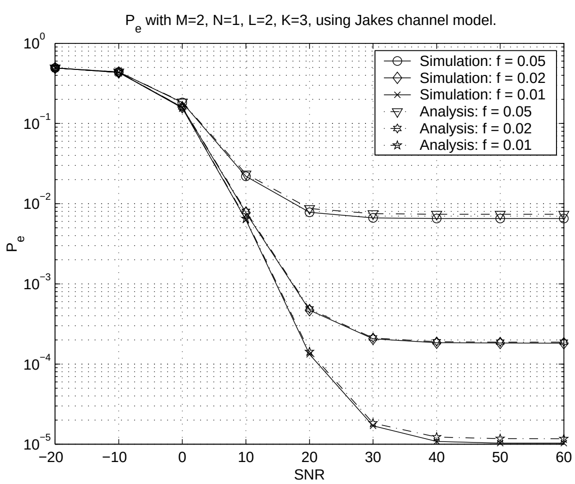

Fig. 4. Results with several speeds of Jakes channel coefficients.

6. CONCLUSIONS

Previous research in space-time modulation has typically assumed channels that are constant for two or more symbol periods. In this paper, we have examined the performance degradation that results when this assumption is violated. We first obtained high-SNR probability of error measures for both trained and differential modulation under a piecewise constant channel model for the general case where the symbol period is not necessarily equal to the coherence interval. It is interesting to note that for this model, the resulting probability of error expressions do not depend on the number of receive antennas. Comparisons between the results for trained and differential modulation reveals that if the coherence interval is not a multiple of the training period, then differential modulation gives better high-SNR performance. This analysis applies not only when using unitary signal matrices, but to all signals obeying (2).

Auto-regressive modeling of the channel variations allowed us to derive expressions for *effective SNR* (ESNR) that combine the effects of the changing channel and the additive noise into a single scalar value when using unitary signal matrices. The ESNR can be used in place of the noise-only SNR to analyze the effects of a time-varying channel using expressions derived assuming the channel to be constant. Comparing ESNR expressions for trained and differential modulation, we are able to determine the signal power that is required in order for differential modulation to outperform trained modulation. Using probability of symbol error as our metric, we validated our analysis with several simulations.

Future areas of research include an investigation of the capacity obtained with these two channel models. Work on capacity so far has focused on the case were the length of a symbol is equal to the coherence interval [8]. Allowing the coherence interval and symbol length to be different would change these results, as would the use of the AR channel model assumed in this paper. A generalization of the piecewise-constant channel with AR variation to one which allows the channel to change at each time sample would also be useful for situations with a large number of antennas (where a longer coherence time is required) as well.

7. REFERENCES

[1] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications*, vol. 6, pp. 311–335, 1998, Available from http://www.bell-labs.com/project/blast/.

[2] I. E. Telatar, "Capacity of multi-antenna gaussian channels," *European Transactions on Telecommunications*, vol. 10, no. 6, pp. 585–595, Nov/Dec 1999, Available from http://mars.bell-labs.com/.

[3] B. M. Hochwald and T. L. Marzetta, "Unitary space-time modulation for multiple-antenna communications in rayleigh flat fading," *IEEE Transactions on Information Theory*, vol. 46, no. 2, pp. 543–564, March 2000, Available from http://mars.bell-labs.com/.

[4] B. L. Hughes, "Differential space-time modulation," *Wireless Communications and Networking Conference*, 1999, Journal version to appear in IEEE Transactions on Information Theory.

[5] B. M. Hochwald and W. Sweldens, "Differential unitary space-time modulation," *To appear in IEEE Trans. Comm.*, March 1999, Available from http://mars.bell-labs.com/.

[6] V. Tarokh and H. Jafarkani, "A differential detection scheme for transmit diversity," *IEEE Journal on Selected Areas in Communications*, vol. 18, no. 3, pp. 1169–1174, July 2000, See http://www.research.att.com/ tarokh/.

[7] I. Korn, "Error floors in the satellite and land mobile channels," *IEEE Transactions on Communications*, vol. 39, no. 6, pp. 833–837, June 1991.

[8] T. L. Marzetta and B. M. Hochwald, "Capacity of a mobile muliple-antenna communication link in Rayleigh flat fading," *IEEE Transactions on Information Theory*, vol. 45, no. 1, pp. 139–157, May 1999, Available from http://mars.bell-labs.com/.

[9] W. C. Jakes, *Microwave Mobile Communications*, IEEE Press, 1993.